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Instability Mode Interaction in a Spatially Developing Plane Wake

Hiroshi Maekawa * and Nagi N. Mansour

NASA Ames Research Center, Moffett Field, CA 94035

ABSTRACT The transition mechanism in a plane wake was investigated by means of numerical simulations of instability mode interaction in a spatially developing wake. The incompressible time-dependent $2-D$ Navier-Stokes equations were solved using finite difference method in the streamwise (x) direction, pseudospectral Fourier method in y direction, and a third order Runge-Kutta scheme for time advancement. The mean profiles, intensity of u-fluctuations and energy spectra of numerical results agree well with experimental measurements. The numerical results show not only the generation of a pair of alternating vortices in the non-linear region, but also, the gradual distortion of a double row of vortices in the downstream location and the loss of a deterministic structure.

1. Introduction

The laminar-turbulent transition in a plane wake, as well as jet flow and separated flow, has been a very important and fundamental phenomenon to the understanding of the mechanism of transition of free boundary shear flow. The experimental studies of the transition region of a two-dimensional wake have been carried out extensively by Sato et. al.[1-4]. Sato & Onda[2] reported the instability mode interaction in a plane wake was understood as a non-linear interaction of each mode -the mutual suppression of amplitude- and generation of new mode. On the other hand, Zabusky and Deem[5] calculated time developing wakes using finite-difference method(128×128 points) and showed the existence of a double row of elliptical vortices.

The advancement of fast and large-memory computer has recently led to the use of direct simulation to predict turbulent flows[6]. Computer simulation and laboratory experiments offer two complementary approaches in this research. Computer simulations can calculate directly fundamental flow quantities such as vorticity and pressure, which is difficult to measure accurately in the laboratory. Moreover, numerical simulation allows to study the instantaneous flow dynamics that are crucial to the understanding of the evolving structure which governs the transition flows.

The purpose of this investigation is to present the spatially evolving process of vortical structure in plane wakes oscillated by linear unstable modes. The non-linear region and the beginning of randomness are studied by means of the statistical analysis of numerical results. Experimental one component energy spectra show the clear existence of a wave number mode family as described above. This paper tries to understand the relations between the transition mechanism as shown by experimental measurements and vortical flow structures which can be seen from the numerical results.

2. Mathematical formulaton

2.1 Governing field equations

Sato & Kuriki[1] reported the flows in the non-linear region of a plane wake were approximated by two-dimensional motions and the change of wave-form was gradual. In this work, the two-dimensional coordinate system is used to represent the spatial variation in the fields. The x coordinate represents the streamwise flow direction. The y coordinate represents the direction perpendicular to x. The streamwise extent of the computational domain is finite and

* Visiting from Kagoshima University, Japan

the y extent is infinite in both the positive and negative y directions. The time-dependent incompressible Navier-Stokes equations:

$$\frac{\partial u_i}{\partial t} = \epsilon_{ijk} u_j \omega_k - \frac{\partial P}{\partial x_i} + Re^{-1} \nabla^2 u_i \tag{1}$$

($Re \equiv U b^0_{1/2}/\nu$, $b^0_{1/2}$ represents the half width of the inlet laminar wake flow) are solved. The conservation of mass for the fluid is expressed by the continuity equation:

$$\frac{\partial u_i}{\partial x_i} = 0. \tag{2}$$

2.2 Boundary conditions

A Gaussian profile was chosen for an inlet flow condition, namely, the mean u component of velocity at the inlet plane is represented by:

$$u = 1 - 0.692 \exp(-0.69315 y^2) \tag{3}$$

The eigenfunction perturbations are invoked in the inlet plane. The unstable eigefuctions of the shear layer problem for the Gaussian profile given by the Eq.(3) were calculated. The fundamental mode, first and second subharmonics are superimposed on the velocity profile at the inlet plane. These perturbations are of the form:

$$u^p_j = \frac{1}{2}\left[\tilde{u_j}(x, y) e^{i\omega_p t} + \text{complex conjugate}\right] \tag{4}$$

The oscillation amplitude was 3%. A time dependent advection condition of the form:

$$\frac{\partial u_i}{\partial t} + U_a \frac{\partial u_i}{\partial x} = 0 \tag{5}$$

is invoked for each of the velocity components at the exit plane, where U_a represents the advection speed of the large-scale structures in the layer. No-stress conditions are employed to represent the boundary conditions at the free stream. That is,

$$\frac{\partial u}{\partial y}\Big|_{y \to \pm\infty} = v = 0. \tag{6}$$

2.3 Initial conditions

The Gaussian profile prescribed for the mean u component at the inlet plane is distributed uniformly at all x locations in the domain at $t = 0$. This profile is perturbed with eigenfuctions which oscillate sinusoidally in time, but only at the inlet plane. These initial conditions must be allowed to wash out before any statistical analysis may be performed on the layer.

3. Numerical formulation

In the present work, the geometry and boundary conditions are such that the finite Fourier transform may be employed in the y direction. The discrete Fourier transform with the FFT algorithm provides relatively high accuracy per degree of freedom.

3.1 y direction representations

A mapping is employed to bring the doubly-infinite extent of the y domain into an interval of finite extent in the computational coordinate, ζ. That is

$$y = -a\cot(2\pi\zeta), \tag{7}$$

where a is a stretching parameter for the mapping. Derivatives in y-space are transformed to ζ-space using the standard chain rule. The procedure is presented in Cain, et. al.[7].

3.2 Time discretization

Third-order Runge-Kutta methods was employed for the time advancement. This scheme allows the flow of high Reynolds number to be integrated in time with a moderately large time step without becoming unstable. Moreover, the scheme can be constructed so that only two words of storage are required per dependent variable.

3.3 Other numerical elements

The finite difference scheme chosen in the streamwise direction was the second-order accurate upwind representation for $\partial/\partial x$. The second-order, central difference scheme was employed to represent the $\partial^2/\partial^2 x$ terms. The discrete form of the poisson equation for the pressure field was solved(see Lowery & Reynolds[8]).

4. Results and discussion

Two cases were studied in the present work: the first (Case 1) is a wake flow forced by three modes; a fundamental mode and its first and second subharmonic. The second (Case 2) is a wake flow forced with a fundamental mode only. In either case, the Reynolds number was 600. In the streamwise direction ($0 \leq x \leq 200$) 512 uniformly distributed grid points were used, 128 grid points were used in the cross-stream direction. In agreement with experimental observation, the numerical mean profiles show the "overshoot" phenomena in the non-linear region. The maximum central value of mean flow is $U_c = 0.838$ at $x = 50$, which is very close to the experimental value of 0.84[1]. Figure 1 shows the distribution of the u-fluctuation at several streamwise locations. The maximum intensity of u-fluctuation is 0.12 at x=25. The fluctuation intensities peak before x=50. Figure 2 shows the instantaneous vorticity contours for two cases. The fundamental mode is the dominant growing mode which saturates into a pair of alternating vortices. Figures 2 (b) and (c) show the flows of the downstream location, $100 \leq x \leq 200$. The distortion of the vortex street in the downstream location can be observed in case 1, while case 2 shows an unperturbed vortex street. Moreover, Fig.2 (b) shows the vortex pairing dynamics around $x = 170$. The large distortion of the vortical structure increases the intensity of u-fluctuation near the wake center shown in Fig.1. In case 1, the structure in the pairing region has various distorted shapes. This is an indication that subharmonics will play a critical role in generating randomness due to shape distortion. Figure 3 shows the velocity fluctuations of u at the location of the peak intensity for case 1. Figure 4 shows the energy spectra of these traces. There exist the resonant higher harmonic components, as well as fundamental, first and second subharmonic components.

5. Conclusion

Numerical results show the vortical structures and energy spectra in a plane wake. In the 2-D case, the distortion of large-scale structure generates the randomness in the flow field.

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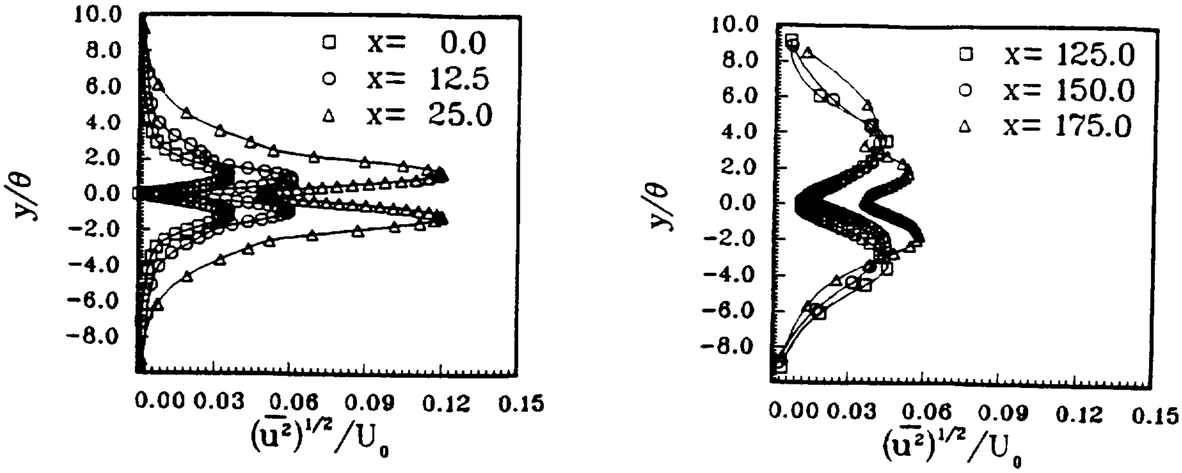

Fig.1 Distributions of u-fluctuation intesity

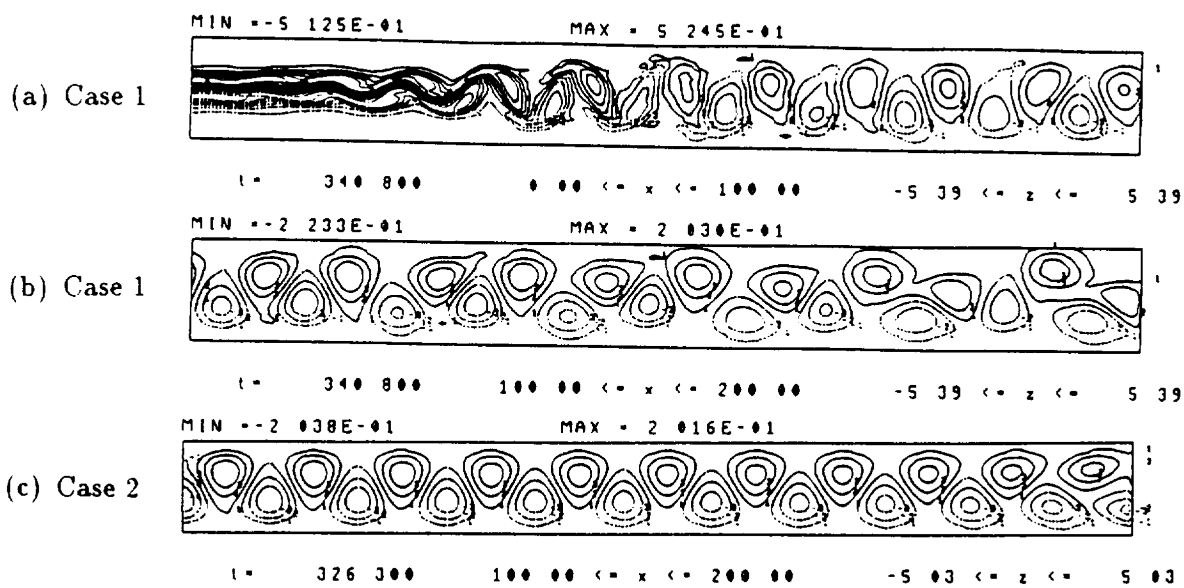

Fig.2 Vorticity contours

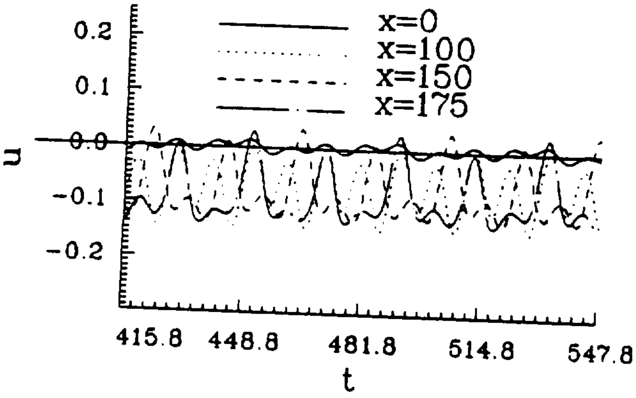

Fig.3 Time trace of u-fluctuation

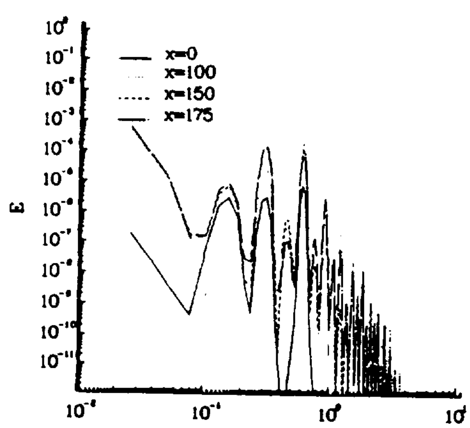

Fig.4 Energy spectra of u